Experiment No. 10

MATLAB Implementation of Fourier Series

Objectives

1. To learn about the Fourier series and its implementation using MATLAB.
2. To understand the effects of Fourier co-efficient on a signal.

Equipment

1. PC
2. Installed MATLAB

Introduction

A Fourier series is an expansion of a periodic function in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical.

Fourier Series of Periodic Signals

We define a continuous-time signal x(t) to be periodic if there is a positive

$$x(t+T) = x(t) \qquad \forall\, t$$

The fundamental period T_o of $x(t)$ is the smallest positive value of T is satisfied, and $\frac{1}{T_0} = f_0$ is referred to as the *fundamental frequency*. An example of periodic signals is the real sinusoidal signal:

$$x(t) = \cos\left(\omega_0 t + \emptyset\right)$$

Where $\omega_0 = 2\pi f_0$ is called the *fundamental angular frequency*.

Fourier series representation of a periodic signal $x(t)$ with fundamental period T_0 is given by:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

where a_0 a_n , b_n are the Fourier coefficients given by:

$$a_0 = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)dt$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)cos(n\omega_0 t)dt$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)sin(n\omega_0 t)dt$$

Activity-1: Implement a square wave signal using Fourier series in MATLAB. Compute five Fourier co-efficients. Display the Fourier series approximation for 5-time units. Change the number of co-efficients and check its effect on the Fourier approximation.

```
clc
close all
clear
fs=100;
T=2;
w0=(2*pi)/T;          %w0=2*pi*f*t, where f=1/T
k=0:1/fs:5-(1/fs);
x=square(w0*k);       %x=sqaure(2*pi*f*k), where f=1/T
syms t
N=5;                  % Number of Coefficients
a0=(2/T)*(int(1,t,0,1)+int(-1,t,1,2));
n=1:N;
an=(2/T)*(int(1*(cos(n*w0*t)),t,0,1)+int(-1*(cos(n*w0*t)),t,1,2));
bn=(2/T)*(int(1*(sin(n*w0*t)),t,0,1)+int(-1*(sin(n*w0*t)),t,1,2));
y=a0/2;
for i=1:N
    y=y+(an(i)*cos(i*w0*k))+(bn(i)*sin(i*w0*k));
end
figure, plot(k,x);
hold on
plot(k,y),title('Fourier Series Implementation Using Square
Wave');
grid on;
legend('Original Signal: x(t)','Signal generated using Fourier
series');
```

What Is the Gibbs Phenomenon?

The Gibbs phenomenon is the persistent overshoot (ringing) that appears near a jump discontinuity when a signal is approximated using a finite number of Fourier series terms (finite N).

What Happens

- The approximation overshoots the true value just before and after the jump.
- It oscillates (rings) around the flat regions near the discontinuity.
- The overshoot height stays at a fixed percentage (≈ 8.9%) of the jump size, no matter how many terms N are added.
- As N increases, the width of the ringing region shrinks (squeezes closer to the jump), but the peak height of the overshoot does not shrink to zero.

Why It Happens

A discontinuity requires infinitely many frequency components to represent perfectly. Truncating the series at any finite N leaves out the high-frequency content needed to sharpen the edge - the missing information manifests as oscillatory ringing, similar to how a low-pass filter causes ringing near sharp edges.

Illustration - Square Wave Approximation

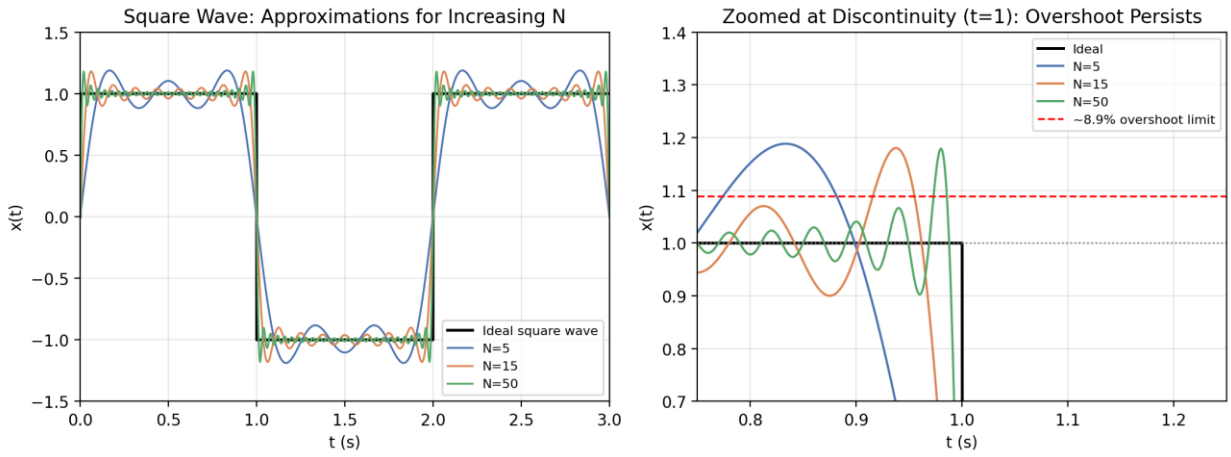


Figure 10.1: Illustration - Square Wave Approximation

Left: *approximations for N = 5, 15, 50 overlaid on the ideal square wave - the ripple narrows as N grows.*

Right: *zoomed view at the discontinuity (t = 1) - the overshoot peak stays pinned near the 8.9% line even at N = 50.*

Key Takeaway for the Lab

N (harmonics)	What changes
Small N	Wide, visible overshoot near the jump

N (harmonics)	What changes
Large N (even N=1000)	Overshoot narrows in width, height stays ≈ 8.9% of jump
N → ∞	Overshoot width → 0, but height never disappears

This is exactly what Task 3 (square wave, N = 1, 5, 15, 50) is designed to demonstrate: students should see the ripple shrink in width across the four plots, but the measured overshoot percentage should hover around 8.9% even at N = 50, proving it does not vanish.

1. Theory Recap

Trigonometric Fourier Series (real form), for a periodic signal x(t) with period T and fundamental frequency $\omega_0 = 2\pi/T$:

$$x(t) = a_0 + \Sigma_n[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

$$a_0 = (1/T)\int x(t)\, dt$$

$$a_n = (2/T)\int x(t)\cos(n\omega_0 t)\, dt$$

$$b_n = (2/T)\int x(t)\sin(n\omega_0 t)\, dt$$

Useful Symmetry Shortcuts

Signal symmetry	Result
Even function: x(−t) = x(t)	$b_n = 0$ (cosine series only)
Odd function: x(−t) = −x(t)	$a_n = 0$ (sine series only)
Half-wave symmetric: x(t+T/2) = −x(t)	Only odd harmonics exist

Complex Exponential Form

$$x(t) = \Sigma_n c_n e^{\wedge}(jn\omega_0 t),$$

$$c_n = (1/T)\int x(t)\, e^{\wedge}(-jn\omega_0 t)\, dt$$

Relation to trig coefficients: $c_0 = a_0$, $c_n = (a_n - jb_n)/2$ for $n > 0$.

2. Key MATLAB Commands Reference

Command	Purpose
linspace(a,b,N)	Generate time vector with N points
square(w0*t)	Built-in ideal square wave (±1)

Command	Purpose
sawtooth(w0*t)	Built-in ideal sawtooth wave
sawtooth(w0*t,0.5)	Built-in ideal triangular wave
for n = 1:N	Loop over harmonics
for n = 1:2:N	Loop over odd harmonics only
subplot(rows,cols,idx)	Multiple plots in one figure
stem(x,y)	Discrete plot — spectrum / coefficients
integral(@(t) f(t), a, b)	Numerically evaluate Fourier integrals
semilogy(x,y)	Log-scale y-axis — MSE / error plots
legend(...,'Location','best')	Auto-place legend

3. General-Purpose Template: Reconstructing Any Periodic Signal

This is the core pattern every lab task reuses - understand it before customizing for a specific signal.

```
%% General Fourier Series Reconstruction Template
T   = 2;                    % period of the signal
w0  = 2*pi/T;               % fundamental frequency
t   = linspace(0, 5, 2000);     % time vector
N   = 10;                   % number of harmonics
x_approx = a0 * ones(size(t));    % start with DC term
for n = 1:N
    an = ...;               % formula for a_n (derived by hand)
    bn = ...;               % formula for b_n (derived by hand)
    x_approx = x_approx + an*cos(n*w0*t) + bn*sin(n*w0*t);
end
plot(t, x_approx);
```

Tip: If the signal has only odd harmonics (half-wave symmetry), change the loop to "for n = 1:2:N".

4. Quick-Reference Formulas for Common Signals

Signal	Period	Fourier Series
Square wave (±1, 50% duty)	T	$(4/\pi)\ \Sigma(n \text{ odd})\ \sin(n\omega 0t)/n$
Sawtooth (rising ramp)	T	$(2/\pi)\ \Sigma\ (-1)^{(n+1)}\ \sin(n\omega 0t)/n$
Triangular wave	T	$(8/\pi^2)\ \Sigma(n \text{ odd})\ (-1)^{((n-1)/2)}\ \sin(n\omega 0t)/n^2$
Full-wave rectified sine	T	$2/\pi - (4/\pi)\ \Sigma\ \cos(2n\omega 0t)/(4n^2-1)$

5. Measuring Overshoot (Gibbs Phenomenon) - Reusable Code Block

```
%% Reusable overshoot-measurement snippet
idx_region  = (t > t_jump) & (t < t_jump + 0.3);   % window after discontinuity
peak_value  = max(abs(x_approx(idx_region)));
overshoot_pct = (peak_value - amplitude) / (2*amplitude) * 100;
fprintf('Overshoot at N=%d: %.2f%%\n', N, overshoot_pct);
```

Expected limiting value: ≈ 8.9% of the jump height, regardless of N (the Gibbs constant).

6. Common Student Errors to Watch For

- Off-by-one in odd harmonics: using 1:N instead of 1:2:N for half-wave symmetric signals adds unwanted even harmonics.
- Wrong ω0: forgetting $\omega 0 = 2\pi/T$ and using $\omega 0 = 2\pi$ regardless of the actual period.
- Coarse time vector: too few points in linspace makes the overshoot peak look smaller than it really is - always use ≥1000-2000 points for Gibbs analysis.
- MSE not normalized: comparing MSE across signals of different amplitude without normalizing gives misleading convergence comparisons.

Lab Task (Psychomotor)

1. Generate the given periodic signal with period $T = 3$ using MATLAB. Display this signal and its Fourier series approximation with 10 coefficients (N=10) in the same figure using $0 \leq t \leq 5$.

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 2 & 1 \leq t \leq 2 \\ 3 & 2 \leq t \leq 3 \end{cases}$$

2. Using the same signal $x(t)$ from Task 1, compute and plot the Fourier series approximation for N = 1, 3, 5, 10, 25, and 50 coefficients. Use a subplot (2×3 grid) to display all six approximations against the original signal on the same axes. Comment on how the approximation improves as N increases, and note the persistent overshoot near the discontinuities (Gibbs phenomenon).
3. Generate a square wave of period T=2 (amplitude ±1, 50% duty cycle) using its Fourier series with only odd harmonics, for N = 1, 5, 15, 50. Plot all four approximations overlaid with the ideal square wave over $0 \leq t \leq 6$. Measure and report the percentage overshoot at the discontinuity for each N, and verify it does not vanish even as N→50 (Gibbs phenomenon).

Lab Task (Cognitive)

Q. No.	Cognitive Assessment Question	Marks
1	For the periodic signal of period T=3 given in the manual, determine its fundamental frequency f0.	2
2	The signal x(t)=1 on $0 \leq t \leq 1$, 2 on $1 \leq t \leq 2$, and 3 on $2 \leq t \leq 3$ is repeated every 3 seconds. Explain why a Fourier-series representation can approximate this signal.	2
3	As the number of Fourier-series terms changes from N=1 to N=50, determine what aspect of the approximation should improve away from discontinuities.	2
4	For the 50% duty-cycle square wave with levels ±1, identify whether even or odd harmonics occur and justify from its symmetry.	2
5	Explain why increasing the number of Fourier-series terms does not eliminate the overshoot near a jump discontinuity, even though the approximation becomes more accurate elsewhere.	2

Lab Exercise and Summary

Summary should cover Introduction, Procedure, Data Analysis and Evaluation.

Student's Signature: _____ **Date: _____**

LABORATORY SKILLS ASSESSMENT (Cognitive)

Obtained Marks	/10

Instructor's Signature: _____ Date: _____

LABORATORY SKILLS ASSESSMENT (Psychomotor)

Psychomotor Activity 1 — Experimental Setup

S. No	Criteria	Performance	Description	Marks
1	**Fourier-Series MATLAB Setup (4)**	Level 4	Correctly prepares MATLAB and the time-domain environment for periodic-signal reconstruction.	4
		Level 3	Correctly prepares MATLAB with minor errors or assistance.	3
		Level 2	Partially prepares the required environment and requires considerable assistance.	2
		Level 1	Shows limited ability to prepare MATLAB and requires substantial assistance.	1
2	**Fourier-Series File Setup (4)**	Level 4	Correctly organizes the script for coefficient calculation, harmonic summation and multiple N values.	4
		Level 3	Completes the file setup with minor errors or assistance.	3
		Level 2	Partially completes the file setup and requires considerable assistance.	2
		Level 1	File setup is largely incomplete and requires substantial assistance.	1
3	**Period & Harmonic Parameters (2)**	Level 2	Correctly defines T, fundamental angular frequency, time vector and harmonic counts.	2
		Level 1	Defines T, fundamental angular frequency, time vector and harmonic counts with errors or assistance.	1
Total Obtained				/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Psychomotor Activity 2 — Implementation + Results

S. No	Criteria	Performance	Description	Marks
1	**Fourier-Series Implementation & Safe Working Practice (4)**	Level 4	Correctly implements the Fourier-series coefficient/summation procedure for the assigned signal independently and follows proper laboratory practices.	**4**
		Level 3	Implements the assigned work with minor errors or assistance and generally follows proper laboratory practices.	**3**
		Level 2	Partially implements the assigned work and requires considerable assistance or repeated guidance.	**2**
		Level 1	Implementation is largely incomplete, requires substantial assistance, or shows poor adherence to proper laboratory practices.	**1**
2	**Execution & Approximation Generation (2)**	Level 2	Correctly executes the code and generates approximations for the required harmonic counts	**2**
		Level 1	Executes the code and generates approximations for the required harmonic counts with errors or assistance.	**1**
3	**Fourier Results & Gibbs Verification (4)**	Level 4	Produces correct reconstructions and verifies approximation improvement and Gibbs overshoot.	**4**
		Level 3	Produces mostly correct reconstructions and verifies approximation improvement and Gibbs overshoot with minor errors or omissions.	**3**
		Level 2	Produces partially correct reconstructions and verifies approximation improvement and Gibbs overshoot with noticeable errors or omissions.	**2**
		Level 1	Results/verifications are largely incorrect or incomplete.	**1**
	Total Obtained			/10

If a student is absent:

Criteria	Description	Marks
Absent	Student is absent from the laboratory activity.	**0**

Instructor's Signature: _____ **Date:** _____

LABORATORY SKILLS ASSESSMENT (Affective)

Affective Activity — Lab Report + Viva Voce

S. No	Criteria	Performance	Description	Marks
1	**Lab Report Completeness & Organization (4)**	Level 4	Report is complete, well-organized, properly presented, and demonstrates responsible completion of the assigned Signals and Systems laboratory work.	**4**
		Level 3	Report is complete with minor omissions or organizational issues and demonstrates satisfactory responsibility for the assigned work.	**3**
		Level 2	Report is partially complete with several omissions and demonstrates limited responsibility for the assigned work.	**2**
		Level 1	Report has major omissions or poor organization and demonstrates minimal responsibility for the assigned work.	**1**
2	**Code, Results & Conclusion (2)**	Level 2	MATLAB code, results/plots, and conclusion are correctly presented and discussed, demonstrating ownership of the assigned work.	**2**
		Level 1	MATLAB code, results/plots, or conclusion are presented with minor errors or omissions.	**1**
3	**Viva Voce (4)**	Level 4	Clearly explains the assigned MATLAB code, signal/system concepts, analysis, results, and plots, demonstrating individual responsibility, participation, and effective communication.	**4**
		Level 3	Explains most of the assigned work and results correctly, with minor difficulty, and demonstrates good individual participation.	**3**
		Level 2	Explains basic aspects of the assigned work with some prompting and demonstrates basic participation.	**2**
		Level 1	Demonstrates limited understanding or involvement and requires substantial prompting to explain the assigned work.	**1**
	Total Obtained			/10

The student has not submitted the report and has not appeared in the Viva Voce.

Criteria	Description	Marks
Lab Report and Results Not Submitted	Student has not submitted the report and results file.	**0**
Absent in Viva Voce	Student is absent from the viva voce.	**0**

Instructor's Signature: _____ **Date:** _____